647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2023(Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any ONE. **[07]**

(i) Explain construction of Cantor set on [0, 1]. Show that a cantor set is closed. What is the length of Cantor set on [0, 1] ? Justify.

(ii) Show that a finite intersection of open sets is open in a metric space. What can you say about finite union of open sets, in this regard?

(B) Answer any ONE. **[04]**

(i) Prove that a finite subset of any metric space is closed.

(ii) Consider a discrete metric space on the set of real numbers R. Now find δ-neighbourhood of point $2 \in R$ i.e. $N(2, \delta)$ for $\delta = 0.75, 1$ and 1.25.

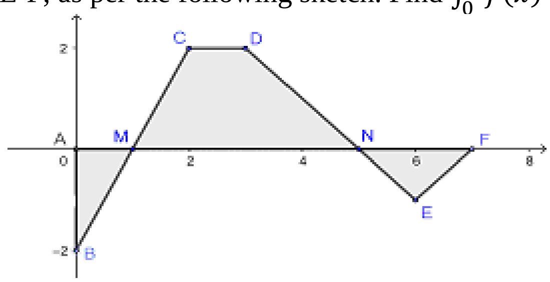
(C) Answer any ONE. **[03]**

(i) Define interior point, derived set and limit point.

(ii) Is the set of integers, dense in the set of real numbers? Justify.

Q.2 (A) Answer any ONE. **[07]**

(i) Let the function $f: [0,7] \to [-3,3]$ be defined by the line-graph given below i.e. graph of f is traced as B-M-C-D-N-E-F, as per the following sketch. Find $\int_0^6 f(x)\,\mathrm{dx}$ and $\int_0^6 |f(x)|\,\mathrm{dx}$.



(ii) Let f be a bounded function on $[a,b]$ and P and P^* be two partitions of $[a,b]$ such that $P \subset P^*$. Then show that $U(P^*, f) \subset U(P, f)$.

(B) Answer any ONE. **[04]**

(i) Evaluate : $\lim\limits_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2\right]$.

(ii) Show that the sum of two Riemann integrable functions on a bounded set $[a,b]$ is Riemann integrable.

(C) Do as directed. Answer any THREE. **[03]**

(i) Let $f: R \to R$ be a bounded periodic function with principle period = 5 and $\int_3^8 f(x)\,\mathrm{dx} = \frac{3}{7}$ then find the value of $\int_{11}^{16} f(x)\,\mathrm{dx}$.

(ii) State the first mean value theorem for Riemann integration.

(iii) Product of two R-integrable functions is R-integrable. *True/False.*

(iv) A function with finite number of removable discontinuity points is not Riemann integrable. *True/False.*

Q.3 (A) Answer any ONE. **[07]**

(i) If $f \in R[a,b]$, then show that $m(b-a) \leq \int_a^b f(x)\, dx \leq M(b-a)$ where m and M are respectively the infimum and supremum of f on $[a,b]$.

(ii) Prove that the six 2×2 matrices $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ with $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ forms a group under matrix multiplication. Is this group commutative?

(B) Answer any ONE. **[04]**

(i) Solve $\int_0^1 x^2\, dx$ using the definition of Riemann integration.

(ii) Prepare a group table for multiplicative group $(Z_{16}^{*}, \cdot)$ in usual notations. Now list down the inverse of each element of this group.

(C) Answer any ONE. **[03]**

(i) Let $a \in G$ and G be a group. If $o(a) = n$, prove that $o(\text{bab}^{-1}) = n, \forall b \in G$.

(ii) Let G be a group. If $a^2 = e$ for each element $a \in G$, then show that G is a commutative group.

Q.4 (A) Answer any ONE. **[07]**

(i) State and prove Euler's theorem.

(ii) Let $(G,*)$ be a group and $(H,*)$ be a subgroup of G. Show that both will have same identity element and inverse of an element of H will be same in G and H both.

(B) Answer any ONE. **[04]**

(i) Find order of permutation β, if $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 6 & 1 & 4 \end{pmatrix} \in S_6$. Also write it as a product of transpositions.

(ii) If G be a group such that $(\text{ab})^n = a^n b^n$ for 3 consecutive integers n then show that G is an abelian group.

(C) Answer any ONE. **[03]**

(i) What is an alternating subgroup? Illustrate with an example.

(ii) Define a transposition. What is its order? Also give an example.

Q.5 (A) Answer any ONE. **[07]**

(i) Let G be a group and let $g \in G$ be a fixed element of G. Then prove that the mapping $i_g: G \to G$, $i_g(x) = \text{gxg}^{-1}\ \forall x \in G$ is an isomorphism.

(ii) Let G and G' be two groups and if $\varphi: G \to G'$ is an isomorphism of groups then $\varphi^{-1}: G' \to G$ is also an isomorphism.

(B) Answer any ONE. **[04]**

(i) Find $f^{-1}g^{-1}$ and $g^{-1}f^{-1}$ for the following permutations f and g.
$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 4 & 5 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \end{pmatrix}$.

(ii) Let $(G_1,*) \simeq (G_2, \circ)$. Show that G_1 is commutative if and only if G_2 is commutative.

(C) Answer any ONE. **[03]**

(i) If $\mu = (2\ 3\ 5)\ (4\ 6) \in S_6$ then find μ^{-1} .

(ii) Find order of additive cyclic subgroup generated by element 3 of Z_8 .

****